Complex Number 3

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$z_1 = (1, 2) \quad z_2 = (-2, 3) \quad z_3 = (1, -1)$

$z_1 = 1+2i \quad z_2 = -2+3i \quad z_3 = 1-i$

$$\frac{z_1}{z_2} + \frac{z_2}{z_3} + \frac{z_3}{z_1}$$

$z\bar{z} = \underbrace{|z|^2}_{\text{Real}}$

$$\frac{z_1}{z_2} = \frac{(1+2i)(-2-3i)}{(-2+3i)(-2-3i)} = \frac{-2-3i-4i+6}{4+9} = \frac{4-7i}{13}$$

$$\frac{z_2}{z_3} = \frac{(-2+3i)(1+i)}{(1-i)(1+i)} = \frac{-2-2i+3i-3}{2} = \frac{-5+i}{2}$$

$$\frac{z_3}{z_1} = \frac{(1-i)(1-2i)}{(1+2i)(1-2i)} = \frac{1-2i-i-2}{5} = \frac{-1-3i}{5}$$

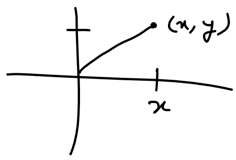
$$\frac{z_1}{z_2} + \frac{z_2}{z_3} + \frac{z_3}{z_1} = \frac{(4-7i)10 + (-5+i)65 + (-1-3i)26}{130} = \frac{\quad}{130}$$

$\overline{z_1+z_2} = \bar{z}_1 + \bar{z}_2 \qquad \overline{\sum z_i} = \sum \bar{z}_i$

$\overline{z_1 \cdot z_2} = \bar{z}_1 \bar{z}_2 \qquad \overline{\prod z_i} = \prod \bar{z}_i$

<u>Properties of Modulus:-</u> $|z| = \sqrt{x^2+y^2} \quad z = x+yi \quad z = (x, y)$

- $-|z| \leq Re(z) = x = \frac{z+\bar{z}}{2} \leq |z|$



$\sqrt{x^2+y^2} = |z| \geq \sqrt{x^2} = x$

$\Rightarrow y^2$ is positive or 0 so $x^2+y^2 \geq x^2$

$y^2 \geq 0$

$\boxed{a \geq b \\ a^2 \geq b^2}$

$\boxed{a^2 \geq b^2 \\ \Rightarrow a \geq b \text{ if } a, b \geq 0}$

- $-|z| \leq Im(z) = y = \frac{z-\bar{z}}{|2i|} \leq |z|$

$|i| = 1$

$|ai| = |a|$

$|z_1 z_2| = |z_1||z_2|$

- $|z| \geq 0 \quad \forall z \in \mathbb{C} \quad |z| = 0$ iff $z = 0$

- $z \cdot \bar{z} = |z|^2$

- $|z_1 z_2| = |z_1||z_2| \quad z_1 = x_1 + y_1 i \quad z_2 = x_2 + y_2 i$

$x_1 \in \mathbb{R}$, $x_2 \in \mathbb{R}$, $y_1 \in \mathbb{R}$, $y_2 \in \mathbb{R}$

$z_1 z_2 = (x_1+y_1 i)(x_2+y_2 i)$

$= x_1x_2 - y_1y_2 + (x_1y_2 + x_2y_1)i$

$$|z_1 z_2| = \sqrt{(x_1x_2 - y_1y_2)^2 + (x_1y_2 + x_2y_1)^2}$$

$$= \sqrt{x_1^2x_2^2 + y_1^2y_2^2 - \cancel{2x_1x_2y_1y_2} + x_1^2y_2^2 + x_2^2y_1^2 + \cancel{2x_1x_2y_1y_2}}$$

$$= \sqrt{x_1^2x_2^2 + x_1^2y_2^2 + y_1^2y_2^2 + x_2^2y_1^2}$$

$$= \sqrt{x_1^2(x_2^2+y_2^2) + y_1^2(x_2^2+y_2^2)}$$

$$|z_1z_2| = \sqrt{(x_1^2+y_1^2)(x_2^2+y_2^2)} = \sqrt{x_1^2+y_1^2}\sqrt{x_2^2+y_2^2} = |z_1||z_2|$$

• $|z_1+z_2| \leq |z_1| + |z_2|$

$$|z_1+z_2|^2 = (z_1+z_2)(\overline{z_1+z_2}) = (z_1+z_2)(\bar{z}_1 + \bar{z}_2) \qquad \bar{\bar{z}}_1 = z_1$$

$$= z_1\bar{z}_1 + z_1\bar{z}_2 + z_2\bar{z}_1 + z_2\bar{z}_2$$

$$= |z_1|^2 + |z_2|^2 + z_1\bar{z}_2 + z_2\bar{z}_1 \qquad \overline{z_1\bar{z}_2} = \bar{z}_1\bar{\bar{z}}_2 = \bar{z}_1 z_2$$

$$= |z_1|^2 + |z_2|^2 + z_1\bar{z}_2 + \overline{z_1\bar{z}_2}$$

$$= |z_1|^2 + |z_2|^2 + 2Re(z_1\bar{z}_2) \qquad a + \bar{a} = 2Re(a)$$

$\boxed{|z_1| = |\bar{z}_1|} = \sqrt{x^2+y^2}$

$= \sqrt{x^2+(-y)^2}$

$z_1 = x_1 + y_1 i$

$\bar{z}_1 = x_1 + (-y_1)i$

$$-2|z_1\bar{z}_2| \leq 2Re(z_1\bar{z}_2) \leq 2|z_1\bar{z}_2|$$

$$|z_1\bar{z}_2| = |z_1||\bar{z}_2|$$
$$= |z_1||z_2|$$

$$|z_1+z_2|^2 = |z_1|^2 + |z_2|^2 + 2Re(z_1\bar{z}_2) \leq |z_1|^2 + |z_2|^2 + 2|z_1||z_2| = (|z_1| + |z_2|)^2$$

$|z_1+z_2|^2 \leq (|z_1| + |z_2|)^2 \qquad |z_1+z_2| \geq 0$

$|z_1+z_2| \leq |z_1| + |z_2| \qquad |z_1| + |z_2| \geq 0$

$2^2 \leq (-3)^2$

$2 \not\leq -3$

HW. $||z_1| - |z_2|| \leq |z_1 - z_2|$

$||z_1| - |z_2|| \leq |z_1 + z_2|$

$|z_1 - z_2| \leq |z_1| + |z_2|$

$z_1^2 = a + bi$

$z_1 = x + yi$ (assume)

$z_1^2 = x^2 - y^2 + 2xyi$

$x^2 - y^2 = a \Rightarrow x^2 - \left(\frac{b}{2x}\right)^2 = a$

$2xy = b$

$y = \frac{b}{2x}$

$z_1 = x + yi \qquad x \in \mathbb{R}, y \in \mathbb{R}$

$z_1^2 = 3 + 4i$

$3 = x^2 - y^2 \qquad 2xy = 4$

$x^2 - \left(\frac{4}{2x}\right)^2 = 3 \qquad x^2 = m$

$x^2 - \left(\frac{2}{x}\right)^2 = 3 \Rightarrow m - \frac{4}{m} = 3 \Rightarrow m^2 - 4 = 3m$

$\Rightarrow m^2 - 3m - 4 = 0$

$$x^2 - \left(\frac{2}{x}\right)^2 = 3 \Rightarrow m - \frac{4}{m} = 3 \Rightarrow m^2 - 4 = 3m$$

$$\Rightarrow m^2 - 3m - 4 = 0$$

$$(m+1)(m-4) = 0$$

$m = -1$ $\quad x^2 = -1$ ✗ not possible

$m = 4$ $\quad x^2 = 4$

$x = \pm 2$ ✓ possible

$$y = \left(\frac{2}{\pm 2}\right)$$

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} \quad z_2 \neq 0$$

HW

$$\left|\sum z_i\right| \leq \sum |z_i|$$

$$\left|\prod z_i\right| = \prod |z_i|$$

Identity:-

HW. $|z_1+z_2|^2 + |z_1-z_2|^2 = 2\left(|z_1|^2 + |z_2|^2\right)$